A VARIATIONAL APPROACH TO COMPUTE THE EQUIVALENT CAPACITANCE OF COAXIAL LINE DISCONTINUITIES

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ABSTRACT

A variational approach is presented which enables both to compute the discontinuity capacitance in coaxial structures and to state its frequency behaviour. The results compare favourably with the ones previously given in literature. Moreover, the method enables to consider structures having different dielectric permittivities at the sides of the discontinuity.

Introduction

The accurate computation of coaxial-line discontinuities is becoming more and more important in order to improve the precision of standards for microwave measurements, to measure dielectric constants of materials, and to design microwave devices such as filters, etc. In past years, other authors have treated this problem using mode-matching techniques to obtain results when uniform dielectrics fill the structure.[1,2] Our work is based on a variational approach, and it enables to treat the more general case in which the left and right-hand sides of the geometrical discontinuity are filled with different dielectric materials. The results obtained are in very good agreement with the ones previously given [1,2]. Moreover, it has been possible to find the functional dependence of the discontinuity admittance versus frequency starting from d.c. up to the cut-off frequency of thefirst higher TM-mode of the structure.

Essentials of the method

Let us consider the structure of fig. 1, which is sufficiently general for our purposes. Whinnery et al. pointed out that the effect of such discontinuity can

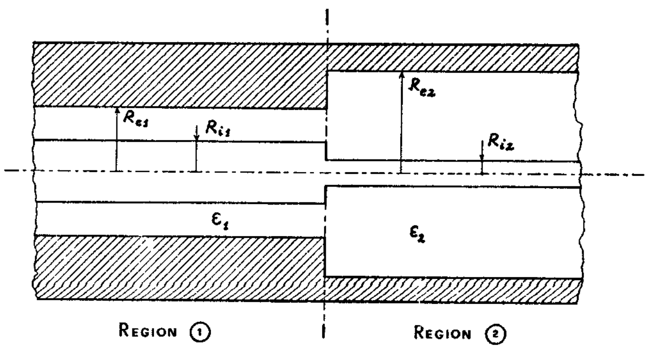

Figure 1. Coaxial Line with Discontinuity

be described through an equivalent transverse capacitance which takes into account the effects of the TM modes arising at both sides of the discontinuity[1]. In order to calculate such capacitance, a variational approach can be developed following the general method given by Collin.[3] The resulting variational relationship is the following:

$$j\omega C = \frac{2\pi\gamma^2}{\ln\left(\frac{R_{e1}}{R_{i1}}\right)} \cdot \frac{\sum_{n=1}^{\infty}\frac{y_{1n}}{\gamma_{1n}^2}\langle E_\rho,\varphi_n\rangle^2 + \sum_{n=1}^{\infty}\frac{y_{2n}}{\gamma_{2n}^2}\langle E_\rho,\psi_n\rangle^2}{\langle E_\rho,\varphi_0\rangle^2} \qquad (1)$$

where:
C is the unknown discontinuity capacitance.
R_{e1}, R_{i1}, R_{e2}, R_{i2} are the geometric dimensions (fig.1)
ε_1, ε_2 are the relative dielectric permittivities.
$\gamma = j\omega\sqrt{\mu\varepsilon_1}$
y_{1n}, y_{2n} are the n-th TM-mode admittances in regions 1, 2 respectively.
γ_{1n}, γ_{2n} are the n-th TM-mode propagation eigenvalues.
φ_n, ψ_n are the n-th TM-mode functions in regions 1, 2 respectively.
φ_0 is the TEM-mode function in region 1.
E_ρ is the radial component of the electric field at the discontinuity.
The brackets denote inner products over the region between R_{i1} and R_{e1}.
Now, the electric field E_ρ may be expanded in terms of the first region mode functions:

$$E_\rho = e_0\varphi_0(\rho) + \sum_{m=1}^{\infty} e_m\varphi_m(\rho) \qquad (2)$$

Inserting eq. (2) into eq. (1), we obtain an expression which should be made stationary with respect to the coefficients e_i (Rayleigh - Ritz method). Actually, the series in (1) and (2) are truncated to proper upper limits. Moreover, C can be calculated minimizing the expression (1) with respect to the coefficients $e_0, \ldots, e_m$ without explicitly obtaining their values.

The functional dependence C(f) of the discontinuity capacitance

As regards the dependence C = C(f), both Whinnery[1] and Somlo[2] gave in the past a certain number of results in graphical form.[1,2] In particular, Somlo stated that it is possible to obtain C at a given frequency multiplying the d.c. value of C by a coefficient K growing with f and depending on the geometric dimensions of the structure.

We have found a good estimate of the functional dependence C(f) as follows. Let us consider again eq. (1), which may be written in the following form:

$$C=\frac{2\pi\gamma^2}{\ln\left(\frac{R_{e1}}{R_{i1}}\right)}\cdot\frac{\frac{1}{j\omega}\left[\frac{y_{11}}{\gamma_{11}^2}\langle E_\rho,\varphi_1\rangle^2+\frac{y_{21}}{\gamma_{21}^2}\langle E_\rho,\psi_1\rangle^2\right]+\left\{\frac{1}{j\omega}\left[\sum_{n=2}^{\infty}\frac{y_{1n}}{\gamma_{1n}^2}\langle E_\rho,\varphi_n\rangle^2+\sum_{n=2}^{\infty}\frac{y_{2n}}{\gamma_{2n}^2}\langle E_\rho,\psi_n\rangle^2\right]\right\}}{\langle E_\rho,\varphi_0\rangle^2} \qquad (3)$$

in the numerator of the fraction, the terms in the first brackets are the ones which first contribute to the growth of C with frequency, while the remaining part gives a contribute which is practically constant at least up to the first cut-off frequency. Under the hypothesis that the first TM-mode cut-off frequency is the one related to the region 2 (otherwise the treatment is somewhat dual of the following), the field E_ρ may be approximated with the first two mode functions related to region 1 :

$$E_\rho \simeq e_0\varphi_0 + e_1\varphi_1 \qquad (4)$$

on the basis of that, imposing the stationarity conditions, straightforward calculations lead to the following functional dependence for C, in which the higher order terms are neglected:

$$C(f) \simeq a + \frac{b}{\sqrt{1-\left(\frac{f}{f_0}\right)^2}} \qquad (5)$$

where a, b are terms depending only on the geometric dimensions and on the dielectric constants $\varepsilon_1, \varepsilon_2$. The term f_0 denotes the first TM-mode cut-off frequency of the structure. This formula is in very good agreement with the data calculated over frequencies from d.c. to f_0 , as is shown in the following results and in each case considered until now.

Results

As regards the calculation of the discontinuity capacitance in d.c., we show results for two cases just considered by Somlo. The tables 1, 2 show our results and the ones obtained by Somlo.[2]

TABLE 1 : R_{e2}= 3mm, R_{i1}= R_{i2}= 1mm, $\varepsilon_1 = \varepsilon_2 = 1$.

R_{e1} (mm)	Our result (fF)	Somlo's (fF)
1.2	70.67	70.64
1.4	46.33	46.12
1.8	22.82	22.23
2.2	10.39	9.64
2.6	3.12	2.63
3.0	0	0

TABLE 2 : R_{e1}= R_{e2}= 3mm, R_{i2}= 1mm, $\varepsilon_1 = \varepsilon_2 = 1$.

R_{i1}(mm)	Our result (fF)	Somlo's (fF)
2.8	207.97	207.93
2.6	135.19	135.08
2.2	65.65	64.94
1.8	29.59	28.27
1.4	8.83	7.82
1.0	0	0

As regards the frequency dependence, we report three cases related to a structure having geometrical dimensions R_{e2}= 5mm, R_{i2}= 2mm, R_{e1}= 4mm, R_{i1}= 2mm. The dielectric permittivity ε_1 has been kept constant and equal to 1, while ε_2 has been given the three values 1, 2, 10. In each case, the cut-off frequency f_0 has been calculated, while the parameters a, b have been obtained through a least-square approximation over twenty points in the whole frequency range considered. The per-cent difference between the calculated and simulated data is in any case lower than 0.95%. Fig. 2

Case 1 :

ε_2 = 1
a = 2.125 (fF)
b = 12.88 (fF)
f_{01} = 49.42 (GHz)

Case 2 :

ε_2 = 2
a = 4.339 (fF)
b = 25.69 (fF)
f_{02} = 34.94 (GHz)

Case 3 :

ε_2 = 10
a = 20.93 (fF)
b = 128.9 (fF)
f_{03} = 15.63 (GHz)

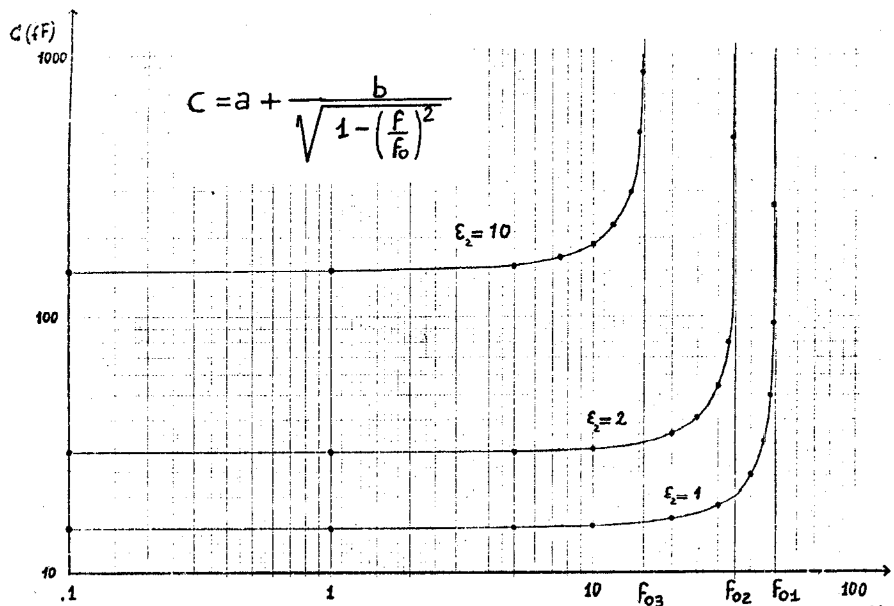

Figure 2. Functional Dependence as a Function of Frequency for E_2 - 1, 2 and 10

shows the three curves, together with the values of a, b, f_0 .

Conclusions

A variational approach has been presented for calculating the discontinuity capacitance in coaxial-line structures with different dielectrics. The method leads to highly accurate results both from the frequency-dependence and the $\mathcal{E}$ -dependence point of wiew. Work is in progress to find the dependence of the parameters a, b on the dielectric permittivities and on the geometrical dimensions.

References

1. J.R. Whinnery, H.W. Jamieson, T.E. Robbins: "Coaxial--Line Discontinuities", Proc. IRE, Vol. 32, Pag. 695 November, 1944.
2. P.I. Somlo: "The Computation of Coaxial-Line Step Capacitances", IEEE Trans. Microwave Theory Techn., Vol. MTT - 15, n. 1, January, 1967.
3. R.E. Collin: "Field Theory of Guided Waves", New York, Mc Graw-Hill, 1960, Ch. 8.